

Stochastic versus Robust Optimization of Wind-Hydro Power Plant's Operational Strategy

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Abstract—In this paper, wind-hydro plant's operational strategy is optimized for obtaining maximum profit. Energy is stored in a water reservoir, which is used accordingly, to maximize the operational profit. An hourly wind forecast of next 48-hours, is used to determine the usage of each component of power plant. Forecasted wind power is stochastic in nature, so two techniques, stochastic programming and Robust Optimization, (RO) are used to determine the operation strategy of wind hydro power plant. It is presented that RO does not require the definition of scenarios or distribution functions for the uncertain data. Moreover, it incorporates uncertainties efficiently without significantly increasing the problem size. Profits estimated by RO are compared with the average profits predicted by Monte Carlo simulations (MCS). Operating schedules obtained from both techniques are also analyzed for further evaluation of each technique in this domain.

Index Terms—Operational strategy, renewable integration, robust optimization, stochastic programming

I. INTRODUCTION

Renewable energy sources have been integrated to the power systems at a very high pace because of environmental sustainability and low toxic emissions associated with them. Fortunately, various governments are encouraging renewable energy sources by providing operational and financial incentives to the power producing companies. Thus, to optimize the operation strategy of such a resource, i.e. to maximize the profits, is of prime interest to the owner.

Technology-wise, wind power plants are getting more advanced day-by-day. However, due to the uncertainty of wind forecast, the output is hard to predict. This makes the dispatch of wind power very difficult, and therefore an auxiliary energy storage system is sometimes used to address this issue. In this work, a water reservoir along with micro-hydro generator works in coordination with wind turbines.

Research in this field has produced a number of options for the controllability of output of wind farms, ultimately resulting in participation of such power plants in energy markets. The daily operational strategy of wind-hydro power plant is investigated in [1], [17] where stochastic characteristics of wind are dealt with Monte-Carlo simulations. Long term economic feasibility is studied in [2], [3] of similar coordinated power plant. Marketing aspects of such power systems are presented in [4] where

trading contracts are exploited. In [5], instead of micro hydro generators, photovoltaic systems and its coordinated operational optimization strategy is deliberated. Storage reserve amount for future operation based on probabilistic wind forecast is discussed in [8], [15] where issue of demand and generation difference is also addressed. For a deterministic wind forecast, operational strategy is optimized in [6]. In [14], integrated approach is used to optimally coordinate wind-hydro plant and an increase of 11% is reported in revenues. Optimal operation strategy for wind-hydro plant for compensation of imbalances is presented in [18]

In this work, a wind hydro facility is considered in which a wind park is augmented with an elevated water storage reservoir, a water-pump and a hydro-generator. This addition results in multifold benefits: 1) to store water in low price periods, so that it can be sold during peak hours, resulting in more profits, 2) to store water during wind storm, making it viable to use in time durations when less wind is available. This water storage will act as a complementary energy source which assists in meeting any contractual commitment with the system operator. This approach helps in increasing daily profit of the wind-hydro facility, in addition to reduction in power output fluctuations which are present due to uneven wind-power profile. The power output is restricted to be in a limit prescribed by network restrictions and contractual obligations. The correlated optimized operation of such power system also results in reduced losses in which best correlation operation is found by Pearson coefficient [16].

It is considered that the wind power forecast for next 48 hours is available and is determined by a methodology described in [7]. Due to random nature of wind, the input data of this problem is subjected to uncertainty. In stochastic programming, probability distribution of the uncertain data is taken into account. But this approach results in huge optimization problems with heavy data requirements.

When some inputs take an uncertain value anywhere between a fixed minimum and a maximum (irrespective of distribution function) RO technique is preferably used. RO is a complementary methodology to stochastic programming and sensitivity analysis [13]. When the input data deviate within the uncertainty range, the feasible solution for all the constraints is obtained. The robustness of decisions is measured in terms of the best performance against all possible realizations of the parameter values. RO is usually

the worst-case oriented methodology. Bidding strategy using RO for wind-hydro plant involving risk measure is determined in [19]. In this paper, RO is used to address uncertainty of wind power and its results are comparatively analyzed with those of MCS.

II. PROBLEM DESCRIPTION

This section gives a description of the physical model, optimization formulation and the system parameters characterizing the wind-hydro power plant.

A. Physical Model

As described previously, the objective is to maximize the profit of a wind-hydro power generation company. For this purpose, the power production of wind farm is controlled to assure power delivery in predefined range. Due to the stochastic wind nature, the standard deviation of the forecasted wind for the next 48 hours is taken into account. To be competitive in the market the wind farm should be able to ensure the energy availability for up to 48 hours ahead and in this way it will increase the gain.

To control the power production of wind farm, a hydro system is incorporated with the wind farm; 1) it consists of a water pump station that elevates water from a source to an upper water reservoir for storage using wind power. 2) A mini hydro generator which can utilize this stored energy to generate power. Instead of using these two machines, reversible hydraulic pump/turbine can be used, which can perform both the pump storage and generation tasks.

Fig. 1 depicts the whole system. It can be observed that there are separate pump and hydro generators integrated with wind generation unit.

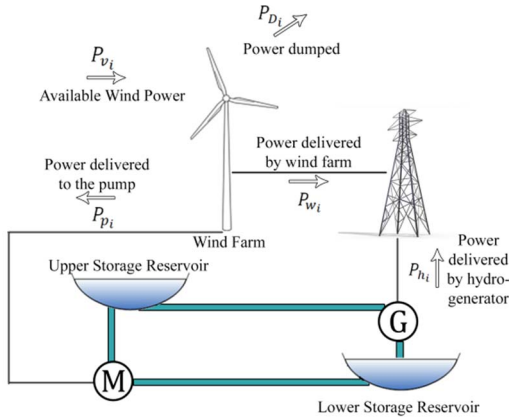


Fig. 1. Wind-hydro power plant overview

The total power delivered to the grid at any instant is the sum of power generated by wind and hydro generator. As per EU directives on promotion of renewable energy [10], such plants are always given priority in their market share by system operator. However, such incentives can cause technical problem, such as congestion of transmission network due to sudden change in wind speed. In order to minimize these problems, such producers are allowed to produce within a certain range, which is predetermined by system operator based on network constraints [1].

B. Optimization Formulation

The optimization formulation for our system is represented by (1) - (13). The solution of these equations

results in the optimized operational strategy providing the maximum operation.

The following optimization problem is solved:

$$\text{Max.}_{(P_i, P_{p_i}, \alpha)} \sum_{i=1}^n (c_i P_i - c_p P_{p_i}) + n c_\alpha \alpha \quad (1)$$

$$\text{s.t.} \quad P_i = P_{w_i} + P_{h_i} \quad (2)$$

$$P_{v_i} = P_{w_i} + P_{p_i} + P_{D_i} \quad (3)$$

$$E_{i+1} = E_i + t(\eta_p P_{p_i} - \frac{P_{h_i}}{\eta_h}) \quad (4)$$

$$E_1 = E_1^{esp} \quad (5)$$

$$E_{n+1} = E_{n+1}^{esp} \quad (6)$$

$$\alpha P_i^L \leq P_i \leq P_i^U \quad (7)$$

$$P_g^L \leq (P_{w_i} + P_{p_i}) \leq P_g^U \quad (8)$$

$$P_h^L \leq P_{h_i} \leq \min(P_h^U, \eta_h \frac{E_i}{t}) \quad (9)$$

$$P_p^L \leq P_{p_i} \leq P_p^U \quad (10)$$

$$0 \leq E_i \leq E^U \quad (11)$$

$$0 \leq \alpha \leq 1.0 \quad (12)$$

$$P_{D_i} \geq 0 \quad (13)$$

Where,

- c : Active power prices vector (hourly)
- P : Active power delivered to the grid
- c_p : Operation cost of pump
- P_p : Average active power consumed by the pump
- c_α : Penalty for generating below the lower output limit
- P_w : Scheduled average active power delivered to the network by wind turbines
- P_h : Hourly active power produced by hydro generator
- P_v : Forecasted hourly available wind power in considered scenario
- P_D : Hourly dumped power
- E : Energy storage levels in the reservoir per hour
- E_1^{esp} : Initial level of the reservoir
- η_p : Water pump and piping network efficiency
- η_h : Reservoir and generator efficiency
- α : Decrease factor in the output power limit
- t : Time duration (in our case its 1 hour)
- E^U : Reservoir storage capacity
- $P_g^{L,U}$: Lower and upper power capacity of wind farm
- $P_p^{L,U}$: Lower and upper power limits as per market requirements
- $P_p^{L,U}$: Lower and upper power limits of pump station
- n : Number of intervals

The objective function consists of two terms; 1) First term is the maximization of the profit by optimizing the power delivered to the grid by wind-hydro plant in active hours. 2) The second term is to meet the requirement of delivering the minimum output power to the grid. When wind power plus hydro power from stored energy is below the essential lower limit, the lower limit constraint in the optimization problem is reduced using $\alpha < 1.0$. The value of α is equal to 1 when the combined wind-hydro operation can produce the minimum power required by market contract.

The active power (2) delivered to the grid comprise of power generated by hydro and wind. In the considered interval, a part of the available wind power (3) is supplied to the grid and some portion can be stored using pump storage which can be delivered in subsequent intervals. Sometimes it may happen that a part of wind energy could not be used.

From (4) the energy balance in reservoir can be calculated. The energy in (i+1)-interval is equal to the energy in i-interval plus the energy pumped minus the energy supplied to the grid by hydro generation in the considered interval. As described by (5) and (6) energy levels of reservoir for both initial and final should be pre-determined. The final level of the previous day is the initial level for the current day, i.e. the initial level is known. The expected operation strategy to be defined for the next day is required to specify of the final level for the current day. To obtain the reservoir level, the study horizon is set to 48 hours.

The output power range is defined in (7). It is desirable to specify the limits due to network operational restrictions, usually the thermal limit of a branch. It can also be due to contractual limitations resulting from the participation of Wind farm in the daily market or bilateral contracts.

The operational restrictions of wind and hydro generators, storage capacity and pumping units are described in equations (8)-(11). Generation limits of wind and hydro plants are shown in (8) and (9), respectively.

C. System Parameters

System bounds are mentioned in Table I. Reservoir capacity is the measure of volume of water that can be stored in the reservoir and its associated potential energy. There are total 6 wind turbines, each capable of 2 MW, thus resulting in total production capacity of 12 MW. The pump has the consuming capacity of 3 MW at maximum, while hydro generator, as categorized as miniature model, is capable of producing maximum 3 MW. Hydro capacity is set to be 24 MWh meaning that 8 hours of continuous operation of pump will fill the upper reservoir.

Table II, presents the information of certain constants that are assumed to be taken from a real life scenario. Collective efficiency of up to 75% is achieved in recent years of reversible hydraulic pump/turbine. So it is divided evenly for pump and hydro-generator. Cost of running a pump per MWh is also mentioned.

TABLE I. LOWER AND UPPER LIMITS OF POWER PLANT COMPONENTS [1]

Lower Bound	Parameter	Upper Bound
0 MW	Capacity of Wind Turbines ($P_{wi} + P_{pi}$)	12 MW
0 MW	Hydro Generator (P_{hi})	3 MW
0 MW	Pump Capacity (P_{pi})	3 MW
0 MWh	Reservoir Capacity (E_i)	24 MWh

TABLE II. CHARACTERISTICS OF POWER PLANT

Parameter	Values
Efficiency of Pump (η_p)	86%
Efficiency of Hydro Generator (η_h)	86%
Cost/Unit of Pump (c_p)	1.5 €/MWh

As mentioned previously, wind-energy remuneration is described for certain tariffs [11], [12]. These prices are independent of market price. Using these tariffs, price per unit of energy is divided as per timings of day. In proposed algorithm vector c contains the energy price for next 48 hours. Table III has tabulated the energy price of each hour for one day. It is assumed that prices are fixed and are known beforehand. Price per unit of energy is divided into two time durations, i.e. peak and off peak times.

TABLE III. ENERGY PRICE DETAILS

Hours	Price/Unit
00:00-08:00	54 €/MWh
08:00-22:00	103.84 €/MWh

Wind power forecast can be attained by using techniques used in [7], [8], and [9]. Reasonable accuracy of the wind power forecast can be achieved for next 48 hours by using the approach of [7], where error distribution curve can be formed for each time horizon. This approach allows obtaining a Gaussian distribution, resulting in the wind forecast to be represented in its average value and corresponding standard deviation. Alternatively, wind-power forecast can be estimated using wind turbines power curves and wind-speed forecast. Fig. 2 shows the typical forecast that will be used in this paper.

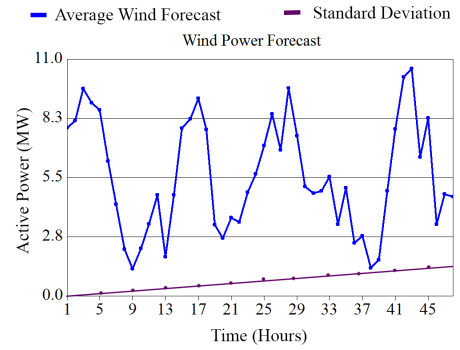


Fig. 2. Wind power forecast: average and standard deviation values

The standard deviation increases with time horizon fostering the reduction in forecast accuracy.

III. STOCHASTIC MODEL OPTIMIZATION

Wind forecast of Fig. 2 is considered which can take any random value within the Gaussian curve defined by its average value and standard deviation for each hour. As it can be seen, that the standard deviation increases with time horizon, therefore the width of curve corresponding to each hour increases accordingly. Adding uncertainty makes the problem closer to real life, since wind uncertainty can impact the optimization results.

The basic idea of Monte Carlo Simulations involves the solving of (1)-(13), and obtain all results of multiple simulations, each using one wind scenario. These results are combined to form an envelope, with significance that most probable operation strategies lay within that envelope. Increasing the number of simulations increases the significance of envelope. In this work, 150 scenarios are generated, i.e. optimization problem is run 150 times. Each time the forecasted wind takes a random value within the limits imposed by the average value and standard deviation.

The presented formulation from (1)-(13) is a linear optimization problem with 289 variables and 241 constraints for each scenario. This problem is solved using CPLEX 12.6 solver in AIMMS using Intel Core i3, 1.7 GHz, 4 GB of RAM. After running the same optimization problems for 150 scenarios, an envelope of operational strategy is obtained. The envelope of active power delivered to the grid by the plant is shown in Fig. 3. The schedules associated with the average profit wind scenario are shown in Fig.4, whereas, pump operational strategy and storage levels for each hour are shown in Fig. 5

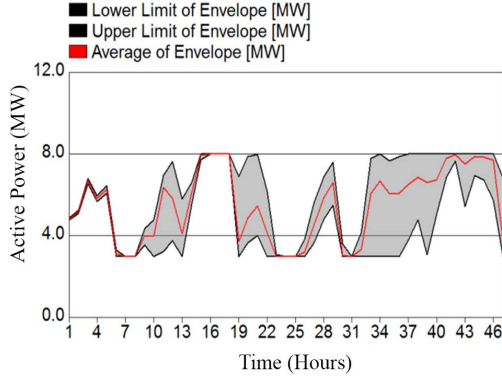


Fig. 3. Envelope of Total Output Active Power of MCS

Within the envelope of Fig.2, there exists a scenario which gives the maximum profit (24303.82 €), minimum profit (22422.81 €), and the average profit (23404.11 €). Notice that the borders of envelope may or may not correspond to any of these values. In Fig. 4, it can be noticed, when the wind generation is unable to meet the minimum power generation limit, power is generated using hydro generator i.e. can be observed from 7th to 11th hour. To maximize the profit the stored energy in the reservoir is used to generate power during the peak hours and energy is stored in off peak hours. It can also be observed that output real power delivered to the system remains in upper (8 MW) and lower bound (3 MW).

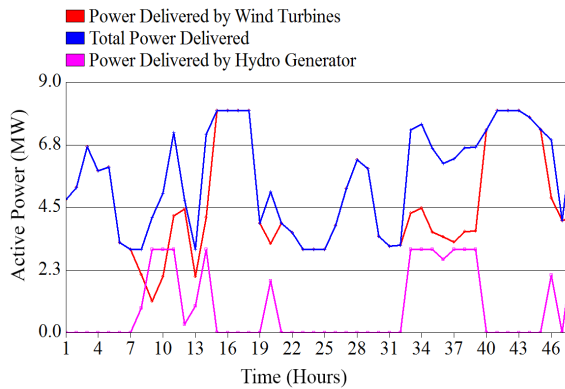


Fig.4. Output power and respective contribution-average of MCS

Fig. 5 shows the energy level in the reservoir and the pump operation for the 48 hour operation strategy. As, it can be observed from the figure that when pump is consuming power (i.e. when pump is 'on' state), the storage level in the reservoir increases, however, it starts decreasing when the hydro plant is producing power.

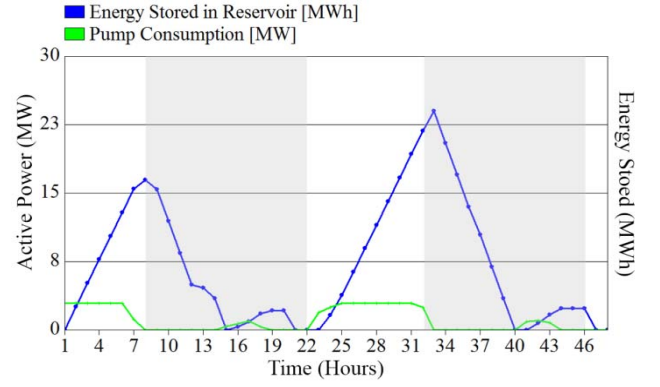


Fig. 5. Reservoir storage and pump operation-average of MCS

IV. ROBUST OPTIMIZATION

Robust optimization is suitable for modeling uncertainties because it does not require the definition of scenarios or distribution functions for the uncertain data. RO adopts a min-max approach that addresses uncertainty by guaranteeing the feasibility and optimality of the solution against all instances of the parameters within the uncertainty set. Therefore, robust optimization incorporates uncertainties by defining region attribute without significantly increasing the problem size. Several predefined regions of uncertainties are available in literature including box, ellipsoid and convex hull.

Using AIMMS Robust Optimization solver, (1)-(13) are solved for available wind of Fig. 3 using ellipsoid uncertainty set represented by (14).

$$\sum_i \left(\frac{P_{v_i} - P_{v_i,level}}{r_i} \right)^2 \leq 1 \quad (14)$$

Contribution of each energy source in case of RO is shown in Fig. 6. Reservoir's energy levels along with pump schedule are represented in Fig. 7.

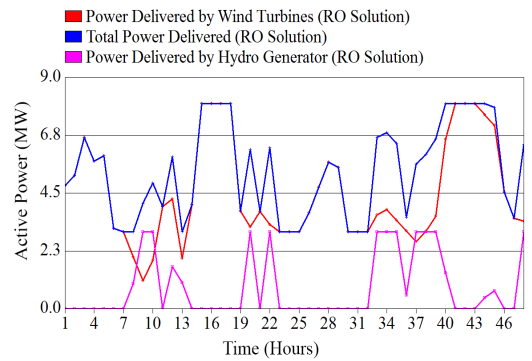


Fig. 6. Output power and respective contribution-RO Solution

The profit associated with RO solution is 22411.43 €.

V. COMPARISON AND DISCUSSION

In this section, results obtained by stochastic programming are compared with RO's results. It is evident that average profit of MCS is higher than that of robust solution. This decrement in profit is directly related to risk aversion. If the plant is run according to the operational strategy obtained by MCS, the chances that the plant run out of energy sources (eventually being heavily penalized due to lowering the

minimum output supply) are higher as compared to the case when it is run using RO schedules.

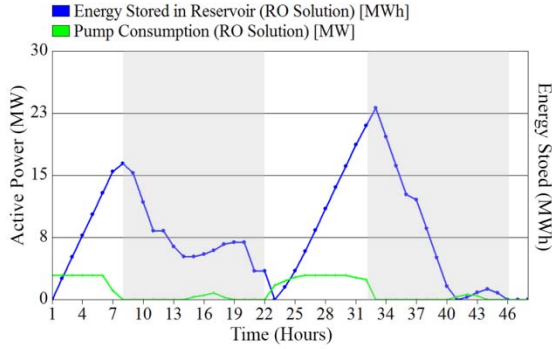


Fig. 7. Reservoir storage and pump operation-RO Solution

For this particular case, consider 19th hour of the day, when according to forecast wind data, 3.95 MW (average) should be available, but actual availability of wind power happens to be much less than anticipated (i.e. $P_{v_i} < 3\text{MW}$). In that case, the plant (considering that it is run based on MCS average schedules) will be unable to meet the market contract, as it has no reserve in the water reservoir, and therefore, it will be heavily penalized ($nc_\alpha\alpha$). On the other hand, operational strategy obtained by RO does have the hydro reserves of 6.9 MWh which could easily compensate for the unanticipated wind deviation and therefore saves the plant from the penalty. Multitudes of such scenarios can be avoided by using RO solution on the cost of profit decrement. This particular case is explained in Table IV.

TABLE IV. ROBUSTNESS EXPLAINED OF RO SOLUTION

	Energy Storage E (MWh)		Power Delivered P (MW)			
	MCS	RO	MCS		RO	
Forecast Wind Power P_{v19} (8.4 MW)	2.1-19 th Hour	6.9-19 th Hour	3.9		3.9	
	0-Consumed	0-Consumed	P_w	3.9	P_w	3.9
	2.1-20 th Hour	6.9-20 th Hour	P_h	0	P_h	0
Actual Wind Power (0.5 MW-assumed)	2.1-19 th Hour	6.9-19 th Hour	2.6		3	
	2.1-Consumed	2.5-Consumed	P_w	0.5	P_w	0.5
	0.-20 th Hour	4.4-20 th Hour	P_h	2.1	P_h	2.5

Another advantage of RO solution is computational ease. In case of stochastic programming, when 150 scenarios are created and optimized, the total number of variables become $289 \times 150 = 43350$, whereas its robust counterpart is solved with total of 337 variables.

VI. CONCLUSIONS

In this paper, operation strategy of combined wind-hydro facility is determined using different optimization techniques. The usage of water storage ability in making the system running smoothly and abided by all constraints is presented. Stochastic wind-power forecasts are used to

determine the maximum profitable operation strategy using stochastic programming and robust optimization. It is observed that part of wind power is consumed by the pump during low price period or network restrictions, whereas, the hydro plant supplies power to meet the lower limit and usually in high price periods to maximize the profits. The results of both techniques are compared, and it is found that although robust optimization gives less profitable operational schedules, but it is more reliable, dependable, involves less risk and computationally more efficient than stochastic programming.

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